

General Certificate of Education
January 2006
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Monday 23 January 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 6 (enclosed)

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Show that the equation

$$x^3 + 2x - 2 = 0$$

has a root between 0.5 and 1.

(2 marks)

- (b) Use linear interpolation once to find an estimate of this root. Give your answer to two decimal places. (3 marks)

- 2 (a) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\int_0^9 \frac{1}{\sqrt{x}} dx$; (3 marks)

(ii) $\int_0^9 \frac{1}{x\sqrt{x}} dx$. (3 marks)

- (b) Explain briefly why the integrals in part (a) are improper integrals. (1 mark)

- 3 Find the general solution, in **degrees**, for the equation

$$\sin(4x + 10^\circ) = \sin 50^\circ$$
 (5 marks)

- 4 A curve has equation

$$y = \frac{6x}{x-1}$$

- (a) Write down the equations of the two asymptotes to the curve. (2 marks)

- (b) Sketch the curve and the two asymptotes. (4 marks)

- (c) Solve the inequality

$$\frac{6x}{x-1} < 3$$
 (4 marks)

5 (a) (i) Calculate $(2 + i\sqrt{5})(\sqrt{5} - i)$. (3 marks)

(ii) Hence verify that $\sqrt{5} - i$ is a root of the equation

$$(2 + i\sqrt{5})z = 3z^*$$

where z^* is the conjugate of z . (2 marks)

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - i$.

(i) Write down the other root of the equation. (1 mark)

(ii) Find the sum and product of the two roots of the equation. (3 marks)

(iii) Hence state the values of p and q . (2 marks)

6 [Figure 1 and Figure 2, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = kx^n$$

where k and n are constants.

Experimental evidence has provided the following approximate values:

x	4	17	150	300
y	1.8	5.0	30	50

(a) Complete the table in **Figure 1**, showing values of X and Y , where

$$X = \log_{10} x \quad \text{and} \quad Y = \log_{10} y$$

Give each value to two decimal places. (3 marks)

(b) Show that if $y = kx^n$, then X and Y must satisfy an equation of the form

$$Y = aX + b \quad \text{(3 marks)}$$

(c) Draw on **Figure 2** a linear graph relating X and Y . (3 marks)

(d) Find an estimate for the value of n . (2 marks)

Turn over for the next question

Turn over ►

- 7 (a) The transformation T is defined by the matrix \mathbf{A} , where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

- (i) Describe the transformation T geometrically. *(2 marks)*
- (ii) Calculate the matrix product \mathbf{A}^2 . *(2 marks)*
- (iii) Explain briefly why the transformation T followed by T is the identity transformation. *(1 mark)*

- (b) The matrix \mathbf{B} is defined by

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

- (i) Calculate $\mathbf{B}^2 - \mathbf{A}^2$. *(3 marks)*
- (ii) Calculate $(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A})$. *(3 marks)*

- 8 A curve has equation $y^2 = 12x$.

- (a) Sketch the curve. *(2 marks)*
- (b) (i) The curve is translated by 2 units in the positive y direction. Write down the equation of the curve after this translation. *(2 marks)*
- (ii) The **original** curve is reflected in the line $y = x$. Write down the equation of the curve after this reflection. *(1 mark)*
- (c) (i) Show that if the straight line $y = x + c$, where c is a constant, intersects the curve $y^2 = 12x$, then the x -coordinates of the points of intersection satisfy the equation

$$x^2 + (2c - 12)x + c^2 = 0 \quad \text{span style="float: right;">*(3 marks)*$$

- (ii) Hence find the value of c for which the straight line is a tangent to the curve. *(2 marks)*
- (iii) Using this value of c , find the coordinates of the point where the line touches the curve. *(2 marks)*
- (iv) In the case where $c = 4$, determine whether the line intersects the curve or not. *(3 marks)*

END OF QUESTIONS

Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											

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Insert

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Insert for use in **Question 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ►

Figure 1 (for use in Question 6)

X	0.60			2.48
Y	0.26			1.70

Figure 2 (for use in Question 6)